Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

SEVERAL HISTORIC PROBLEMS WHICH HAVE NOT YET BEEN SOLVED.

By DR. G. A. MILLER.

Goldbach's theorem affirms that every even number is the sum of two primes, unity being regarded as a prime number. For instance, $2=1+1$, $4=1+3=2+2$, $6=1+5=3+3$, $8=1+7=3+5$, $10=3+7=5+5$, $12=1+11=5+7$, etc. Although this theorem has been known for more than one hundred and sixty years* yet no one has succeeded in either proving or disproving it. Many attempts have been made, especially in recent years, but these have only served to make it more probable that the theorem is universally true. In two of the recent papers† an attempt is made to find some approxmate laws of increase of the number of ways in which an even number can be resolved into the sum of two primes when this even number becomes large. In the latter of these, the author reaches the conclusion that every even number which exceeds 50,000 is the sum of at least 330 different pairs of primes. It should, however, be emphasized that this result is not proved and that it still appears possible that some even number may be found which is not the sum of two primes.

Another theorem which remains unproved and has a still more extensive history than the one noted in the preceding paragraph is due to Fermat and is known under various names. Two of these are, *the last theorem of Fermat* and *the great theorem of Fermat.* It affirms that the equation

$$x^n+y^n=z^n$$

is not satisfied by any integral value of x, y, z whenever $n>2$; and it is equivalent to the theorem that the difference of the nth power of two rational fractions is never unity whenever $n>2$. The Greeks studied this equation for $n=2$ in connection with the rational right triangle. In his edition of Diophantus, Fermat states on the margin of a page that he has found a wonderful proof for the assertion that $x^n+y^n=z^n$ is not satisfied for integral values of x, y, z when $n>2$ but that the margin is too small to contain the proof. A large number of attempts have since been made but no one has been entirely successful although Kummer succeeded in proving it for an infinite number of special values of n. While it is very likely that the theorem is universally true yet it has not been *proved* impossible to find three integral values which satisfy the given equation for large values of n.

In Ball's History of Mathematics, third edition, page 306, the statement is made that it is not known whether there are any values of n to

*It is contained in letters written by Goldbach and Euler in 1742.

†Haussner, Jahresbericht der Deutschen Mathematiker-Vereinigung, Vol. 5 (1896), p. 62; Ripert, L' Intermediaire des Mathematiciens, Vol. 10 (1903), p. 78.

which Kummer's proof does not apply. This is incorrect as such values are not only known but it is also known that the number of such values increases as n increases* so that an infinite number of values of n do not come under Kummer's proof. Unfortunately Ball's History contains a large number of other inaccurate statements.†

As a third unsolved problem with a long history we may mention that no one has yet either proved or disproved the existence of an odd perfect number. A perfect number is one which like $6=1+2+3$ or $28=1+2+4+7+14$ is equal to the sum of its divisors. Euclid observed that $2^{p-1}(2^p-1)$ is a perfect number whenever 2^p-1 is a prime, and it has since been proved that every even perfect number is of this form. Nine such numbers are known. They increase so rapidly with p that their determination for large values of p is very laborious. Descartes thought it possible that odd perfect numbers exist yet none have been found although the subject has received a great deal of attention.

The questions mentioned above relate to properties of natural numbers that are easily understood. A similar problem which has a less extensive history is, whether there is an infinite number of pairs of primes such that the difference of the numbers which constitute a pair is 2. In other words, whether there is no largest prime such that this prime increased by 2 is again a prime. Many similar questions present themselves in number theory, where the very simple and the very difficult theorems seem to lie side by side without exhibiting any external evidence in regard to the class to which they belong.

A problem which has a much less extensive history than the preceding and which can probably be solved more readily is the determination of a six times transitive function which is neither symmetric nor alternating. This is equivalent to the determination of a six-fold transitive group which is neither alternating nor symmetric. In 1861, Mathien published‡ a five-fold transitive function on twelve letters and announced a similar function on twenty-four letters, which he afterwards explained more fully. Although more than forty years have passed since these discoveries no one has extended these results so that we are still ignorant of any six- or seven-fold transitive functions that are neither alternating nor symmetric. The more general question of determining a limit of transitivity for non-alternating and non-symmetric functions of a given degree has received considerable attention during this period and much progress has been made along this line.

One of the most beautiful theorems due to Lagrange affirms that every algebraic number of the second degree may be written in the form of a periodic continued fraction and that every periodic continued fraction is equal to an algebraic number of the second degree. The great importance

*Cf. Kronecker, Vorlesungen ueber Zahlentheorie, 1901, p. 23.

†Cf. On Ball's History of Mathematics, THE AMERICAN MATHEMATICAL MONTHLY, Vol. 9 (1902), p. 280.

‡Mathien, Journal de Mathematiques, Vol. 6 (1861), p. 241.

of this result led men like Jacobi and Hermite to make efforts to prove a similar theorem with respect to algebraic numbers of the third degree but their efforts were not crowned with success. Comparatively little progress has been made towards useful criteria to determine the degree of algebraic numbers, or even towards determining whether a given number is algebraic or transcendental. One of the problems suggested by Hilbert at the International Congress of Mathematicians held at Paris in 1900 is to determine whether a^p, the base being algebraic and the exponent an irrational algebraic number, always represents a transcendental or at least an irrational number.

It should not be inferred that the preceding problems are suggested as very suitable fields for the young investigator. The main object in stating them is to point out to those who may not have good library facilities that some problems relating to very elementary matters still remain unsolved, and, if possible, to encourage some one to acquaint himself with congeniel fields of study where much remains to be done. A clear understanding of the real nature of unsolved historic problems is of great importance to the stvdent since results bearing on such problems are of especial interest. Many problems of this kind are noted from time to time in *L' Intermediaire des Mathêmaticiens* published by Gauthier-Villars, and a set of about twenty very fundamental ones were given by Hilbert at the Congress mentioned in the preceding paragraph. These have been published in the *Bulletin of the American Mathematical Society*, Vol. 8 (1902), p. 437.

NOTE ON A RECENT PROBLEM IN THE AMERICAN MATHEMATICAL MONTHLY.

By R. D. CARMICHAEL, Anniston, Alabama.

The object of this note is to state in a somewhat more general form a proposition in number theory demonstrated on pages 155-156 of Volume XIII of the MONTHLY.

Given that $P^{\delta a}-R^{\delta a}$ *is divisible by* δa, *the necessary and sufficient conditions that the expression* $\dfrac{P^a-R^a}{\delta a(P-R)}$ *shall be integral are:* (1) a *must be divisible by e, the least integer such that* P^e-R^e *is divisible by* a_k, *where for* a_k *is taken in turn the various prime factors of* a *not dividing* $P-R$; (2) δ *is any divisor of* $(P^a-R^a)/a(P-R)$.

The proof is practically identical with that given in the MONTHLY (*l. c.*), except in showing that δ and $I-R$ have no common factor. If δ_1 is such a prime factor, it becomes necessary in the present case to modify the proof that δ_1 must be a factor of a. This is shown as follows: